❖ The Heisenberg's Uncertainty Principle

In quantum mechanical world, the Heisenberg's uncertainty principle (or simply the uncertainty principle) is one of a variety of mathematical inequalities asserting a fundamental limit to the precision with which certain pairs of physical properties of a particle, known as complementary variables or canonically conjugate variables such as position x and momentum p, can be known. The concept was first introduced in 1927, by a German physicist Werner Heisenberg.

The Heisenberg's uncertainty principle states that the more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa.

The formal inequality relating the standard deviation of position Δx and the standard deviation of momentum Δp_x was derived by Earle Hesse Kennard later that year and by Hermann Weyl in 1928:

$$\Delta x.\Delta p_x \geq \frac{h}{4\pi} \tag{37}$$

or

$$\Delta x\ .\ \Delta p_x \geq \hbar/2 \tag{38}$$

Where $\hbar$ is the reduced Planck's constant, which is obviously equal to the Planck's constant divided by 2π. Besides the equation (37), the is also an energy-time uncertainty relation given by W. Heisenberg which states that higher the lifetime of a quantum mechanical state, less uncertain would be the energy value. Mathematically, it can be shown as:

$$\Delta E.\Delta t \geq \frac{h}{2\pi} \tag{39}$$

Where ΔE and Δt represent the uncertainties in the energy and time respectively.

➢ ***Position Momentum Uncertainty***

Among various kinds of uncertainties, the position-momentum uncertainty is one of the popular kind that arises as a consequence of wave-particle duality. In order to understand the relation, we first need to study the effect of wave behavior on the simultaneous measurement of position about x-coordinate and the linear momentum component along the x-axis for a microscopic particle.

Consider a beam of particles traveling with a momentum "p" along the y-direction, and this beam finally strikes a narrow slit of width "w". Now, from the principles of optics, we know that the uncertainty in the position of the particle along x-axis must be equal to the slit width. In other words, as the width of the slit is along x-axis, any particle that strikes the detector must have crossed the Δx region i.e. w, the slit width available. However, we exactly don't know where it does cross from. It could be along the center of the slit, or along a line slightly above or below the central trajectory. Therefore, the slit width ($w = \Delta x$) would be equal to a crossing domain that we are uncertain about. However, a diffraction pattern will be observed in the case of microscopic particles because of their wave-like character. The amplitude of the wave at a particular point on the detector represents the number of the particles reaching that point. Now because of this diffraction, the incident beam does not strike only at the central point O but also at the above and below to it. It means that some particles do reach upward and downward to O, suggesting that the part of their linear momentum is transferred along x-axis also.

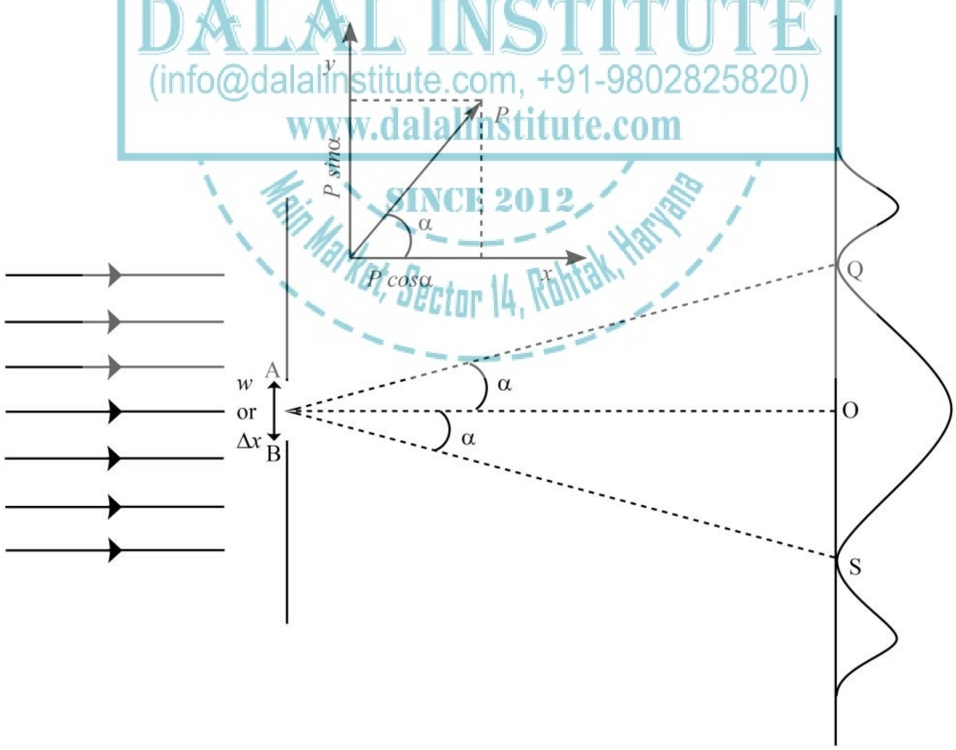

Figure 4. The diffraction of electron waves by single slit systems.

The x-component of linear momentum of the wave (aka particle) diffracted at an angle α can be obtained by the rectangular resolution of the linear momentum vector. The particles diffracted upward and downward at an angle α will yield the x-component as P sinα and $-P$ sinα, respectively. Now because a large number of particles reach the plate in between +α to −α i.e in between the first minimums, half of the momentum spread in the central diffraction peak should give the uncertainty in the momentum along x-axis. Mathematically, we can say that

$$\Delta p_x = P\, sin\alpha \tag{40}$$

Multiplying the above equation by the uncertainty in the position i.e. width of the slit used for the measurement purpose, we get

$$\Delta x.\Delta p_x = w.P\, sin\alpha \tag{41}$$

Here, it is very important to recall the fact that the condition which must be satisfied to obtain the first minima is that the path difference between the waves reaching the minima point should be an integral multiple of λ/2.

Figure 5. The calculation for 1st order diffraction for electron wave in single slit systems.

Hence we have the following equalities from the diagram given above.

$$AQ = DQ \tag{42}$$

$$CQ = difference\ in\ the\ path\ length \tag{43}$$

Now because the distance of the detector is very large as compared to the slit width, AQ and CQ can be considered parallel to each other i.e. AQ ∥ DQ. Hence, we can say that

$$< ADC = 90° \tag{44}$$

$$< CAD = \alpha \tag{45}$$

also

$$AC = \frac{w}{2} \tag{46}$$

$$CD = \frac{\lambda}{2} \tag{47}$$

From the trigonometric relations, we get

$$\frac{CD}{AC} = Sin\ \alpha \tag{48}$$

$$CD = AC\ Sin\ \alpha \tag{49}$$

Putting the values of AC and CD from equation (46) and (47) in equation (49), we get

$$\frac{\lambda}{2} = \frac{w}{2}\ Sin\ \alpha \tag{50}$$

$$\lambda = w\ Sin\ \alpha \tag{51}$$

Now, after putting the value of *w* from equation (51) in equation (41), we get

$$\Delta x. \Delta p_x = \frac{\lambda}{Sin\ \alpha}. P\ sin\alpha \tag{52}$$

$$\Delta x. \Delta p_x = \lambda. P \tag{53}$$

Using the de Broglie relation ($\lambda = h/p$) in equation (53), we get

$$\Delta x. \Delta p_x = \frac{h}{P}. P \tag{54}$$

$$\Delta x. \Delta p_x = h \tag{55}$$

Now because we didn't define the uncertainty very precisely, we should not use the "equal" sign. Therefore, the above equation can be reduced to the following.

$$\Delta x. \Delta p_x \approx h \tag{56}$$

This eventually means that decreasing the uncertainty in the position of the incident particle (decreasing the slit width) would result in a higher uncertainty in the momentum along *x*-axis; while the higher slit width does give more precise momentum but small precision in the calculation of the position of the incident particle.

➢ ***Energy Time Uncertainty***

The uncertainty principle doesn't limit itself to position-momentum only but can also be applied to some other pairs of conjugate variables. All the variable pairs whose products have the same dimension as the Plank's constant h (Js) are said to be a conjugate pair. Besides the position-momentum, another famous uncertainty is relation energy-time because the product of these two quantities (energy × time) also has the unit of h (Js).

$$\Delta E.\Delta t \approx h \tag{57}$$

Where ΔE and Δt are uncertainties in energy and time, respectively. This popular relation can be derived directly from the concept of wave-particle duality. In the quantum mechanical world, a particle is supposed to possess a wave packet. Now, let us consider that this wave packet occupies the Δx region along the direction x-direction and travels with a velocity v. The time it needs to pass a certain point in x-direction has an uncertainty magnitude of Δt, and can be formulated as:

$$\Delta t = \frac{\Delta x}{v} \tag{58}$$

Now because this wave packet occupies the region Δx, the momentum uncertainty along x-axis can be given by the following relation.

$$\Delta p_x = \frac{h}{\Delta x} \tag{59}$$

or

$$\Delta x = \frac{h}{\Delta p_x} \tag{60}$$

Putting the value of Δx from equation (60) in equation (58), we get

$$\Delta t = \frac{h}{v\Delta p_x} \tag{61}$$

Moreover, we also know that

$$E = \frac{p_x^2}{2m} \tag{62}$$

Differentiating the above equation w.r.t p_x, we get

$$\frac{dE}{dp_x} = \frac{\Delta E}{\Delta p_x} = \frac{p_x}{m} = \frac{mv}{m} = v \tag{63}$$

$$\Delta E = \frac{dE}{dp_x}\Delta p_x = v.\Delta p_x \tag{64}$$

Multiplying equation (63) and (64), we get

$$\Delta E.\Delta t = v\Delta p_x . \frac{h}{v\Delta p_x} \tag{65}$$

$$\Delta E.\Delta t \approx h \tag{66}$$

The physical interpretation of the above relation can be viewed in terms of fluctuating energy level with a total ΔE uncertainty if the system does not stay in it longer than Δ*t* interval of time i.e. lifetime of the state.

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Mandeep Dalal
(M.Sc, Ph.D, CSIR UGC - NET JRF, IIT - GATE)
Founder & Director, Dalal Institute
Contact No: +91-9802825820
Homepage: www.mandeepdalal.com
E-Mail: dr.mandeep.dalal@gmail.com

Mandeep Dalal is an Indian research scholar who is primarily working in the field of Science and Philosophy. He received his Ph.D in Chemistry from Maharshi Dayanand University, Rohtak, in 2018. He is also the Founder and Director of "Dalal Institute", an India-based educational organization which is trying to revolutionize the mode of higher education in Chemistry across the globe. He has published more than 40 research papers in various international scientific journals, including mostly from Elsevier (USA), IOP (UK) and Springer (Netherlands) .

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